1. Two Reaction Theory (Blondel’s Theory):

- In salient pole type alternators the length of the air gap varies and the reluctance also varies. Hence the armature flux and field flux cannot vary sinusoidally in the air gap. The reluctances of the magnetic circuits on which m.m.fs act are different in case of salient pole alternators.
- Hence the armature and field m.m.fs cannot be treated in a simple way as they can be in nonsalient pole alternators.
- The theory which gives the method of analysis of the distributing effects caused by salient pole construction is called two reaction theory.

According to this theory the armature m.m.f. can be divided into two components as,

1. Component acting along the pole axis called direct axis Component, it can be magnetising or demagnetising.
2. Component acting at right angles to the pole axis called quadrature axis, it is cross magnetising.

- Let \( F_f \) be the m.m.f. wave produced by field winding, then it always acts along the direct axis. This m.m.f. is responsible to produce an excitation e.m.f. \( E_f \) which lags \( F_f \) by an angle 90°.
- When armature carries current, it produces its own m.m.f. wave \( F_{AR} \). This can be resolved in two components, Similarly armature current \( I_a \) also can be divided into two components, one along direct axis and along quadrature axis.

\[
\begin{align*}
F_{AR} & \rightarrow \begin{cases} 
F_d = \text{Component acting along direct axis} \\
F_q = \text{Component acting along quadrature axis}
\end{cases} \\
I_a & \rightarrow \begin{cases} 
I_d = \text{Component acting along direct axis} \\
I_q = \text{Component acting along quadrature axis}
\end{cases}
\end{align*}
\]

M.M.F. wave positions in salient pole machine
The positions of $F_{AR}$, $F_d$ and $F_q$ in space are shown in the Fig. above. The instant chosen to show these positions is such that the current in phase R is maximum positive and is lagging $E_f$ by angle $\Psi$.

It can be denoted that the reactance offered to flux along direct axis is less than the reactance offered to flux along quadrature axis. Due to this, the flux $\Phi_{AR}$ is no longer along $F_{AR}$ or $I_a$. Depending upon the reluctances offered along the direct and quadrature axis, the flux $\Phi_{AR}$ lags behind $I_a$.

We know that, the armature reaction flux $\Phi_{AR}$ has two components, $\Phi_d$ along direct axis and $\Phi_q$ along quadrature axis. These fluxes are proportional to the respective m.m.f. magnitudes and the permeance of the flux path oriented along the respective axes.

$$\Phi_d = P_d F_d \quad \& \quad \Phi_q = P_q F_q$$

where $P_d \& P_q$ = permeance along the direct axis & quadrature axis.

But $F = \text{m.m.f.} = K_{ar} I$

$$\Phi_d = P_d K_{ar} I_d \quad \& \quad \Phi_q = P_q K_{ar} I_q$$

Where $K_{ar}$ is the armature reaction coefficient.

As the reluctance along direct axis is less than that along quadrature axis, the permeance $P_d$ along direct axis is more than that along quadrature axis, ($P_d < P_q$).

Let $E_d$ and $E_q$ be the induced e.m.f.s due to the fluxes $\Phi_d$ and $\Phi_q$ respectively. Now $E_d$ lags $\Phi_d$ by $90^\circ$ while $E_q$ lags $\Phi_q$ by $90^\circ$.

$$E_d = K_e \Phi_d \angle -90^\circ = -j K_e \Phi_d$$

$$\& E_q = K_e \Phi_q \angle -90^\circ = -j K_e \Phi_q$$

where $K_e$ = e.m.f. constant of armature winding.
The resultant e.m.f. is the phasor sum of \( E_f, E_d \) and \( E_q \).

\[
E_R = E_f + E_d + E_q
\]

Substituting expressions for \( \Phi_d \) and \( \Phi_q \),

\[
E_R = E_f - jK_e \Phi_d - jK_e \Phi_q
\]

Now \( X_{ard} = \text{Equivalent reactance corresponding to the d-axis component of armature reaction} = K_e P_d K_a \)

and \( X_{arq} = \text{Equivalent reactance corresponding to the q-axis component of armature reaction} = K_e P_q K_a \)

\[
E_R = E_f - jX_{ard} I_d - jX_{arq} I_q \quad ... \quad (i)
\]

For a realistic alternator we know that the voltage equation is,

\[
E_R = V_t + I_a R_a + jI_a X_L
\]

where \( V_t = \text{terminal voltage} \)

\( X_L = \text{leakage reactance} \)

**but** \( I_a = I_d + I_q \)

\[
\therefore \ E_R = V_t + I_a R_a + jI_d X_L + jI_q X_L \quad ... \quad ... \quad (ii)
\]

Equating equations (i) & (ii),

\[
V_t + I_a R_a + I_d X_L + I_q X_L = E_f - jX_{ard} I_d - jX_{arq} I_q
\]

\[
E_f = V_t + I_a R_a + jI_d (X_L + X_{ard}) + jI_q (X_L + X_{arq})
\]

\[
E_f = V_t + I_a R_a + jI_d X_d + jI_q X_q
\]

where \( X_d = \text{d-axis synchronous reactance} = X_L + X_{ard} \)

and \( X_q = \text{q-axis synchronous reactance} = X_L + X_{arq} \)

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2. **Detail Analysis of phasor diagram of Two reaction theory:**

Let us find out some geometrical relationships between the various quantities which are involved in the phasor diagram. For this, let us draw the phasor diagram including all the components in detail.

We know from the phasor diagram shown in the Fig. below that,

\[
I_d = I_a \sin \psi \quad ... \quad ... \quad (i)
\]

\[
I_q = I_a \cos \psi \quad ... \quad ... \quad (ii)
\]

\[
\cos \psi = \frac{I_q}{I_a} \quad ... \quad ... \quad (iii)
\]

The drop \( I_a R_a \) has two components which are,
\[ I_d R_d = \text{drop due to } R_a \text{ in phase with } I_d \]
\[ I_q R_a = \text{drop due to } R_a \text{ in phase with } I_q \]

### Phasor diagram for lagging p.f.

In the phasor diagram,

\[ OF = E_f \]
\[ OG = V_t \]
\[ GH = I_a R_a \]
\[ HA = I_d R_a \]
\[ GA = I_a R_a \]
\[ AE = I_d X_d \]
\[ EF = I_q X_q \]

Now DAC is drawn perpendicular to the current phasor \( I_a \) and CB is drawn perpendicular to AE.

- The triangle ABC is right angle triangle,
  \[ \cos \psi = \frac{BC}{AC} = \frac{EF}{AC} = \frac{I_q X_q}{AC} \]

But from equations (iii), \[ \cos \psi = \frac{I_q}{I_a} \]

\[ \therefore \frac{I_q}{I_a} = \frac{I_q X_q}{AC} \]
\[ AC = I_a X_q \]

- Now triangle ODC is also right angle triangle,
  \[ \tan \psi = \frac{CD}{OD} = \frac{CA + AD}{OI + ID} \]
  \[ \tan \psi = \frac{I_a X_q + V_t \sin \phi}{V_t \cos \phi + I_a R_a} \]
As $I_aX_q$ is known, the angle $\Psi$ can be calculated from equation (iv).

Now, from the phasor diagram,

$$\delta = \Psi - \Phi$$

Therefore,

$$\tan \delta = \tan(\Psi - \Phi) = \frac{\tan \psi - \tan \Phi}{1 + \tan \psi \cdot \tan \Phi}$$

$$\tan \delta = \frac{\left[\frac{V_t \sin \Phi + I_aX_q}{V_t \cos \Phi + I_aR_a}\right] - \tan \Phi}{1 + \left[\frac{V_t \sin \Phi + I_aX_q}{V_t \cos \Phi + I_aR_a}\right] \cdot \tan \Phi} = \frac{V_t \sin \Phi + I_aX_q - \tan \Phi \cdot [V_t \cos \Phi + I_aR_a]}{V_t \cos \Phi + I_aR_a + \tan \phi \cdot [V_t \sin \Phi + I_aX_q]}$$

$$\tan \delta = \frac{V_t \sin \Phi + I_aX_q - \frac{\sin \Phi}{\cos \Phi} \cdot [V_t \cos \Phi + I_aR_a]}{V_t \cos^2 \Phi + I_aR_a \cos \Phi + V_t \sin^2 \Phi + I_aX_q \sin \Phi}$$

$$\tan \delta = \frac{I_aX_q \cos \Phi - I_aR_a \sin \Phi}{V_t + I_aX_q \sin \Phi + I_aR_a \cos \Phi} \ldots \ldots \ldots (v)$$

3. Power developed in salient pole synchronous generator:

It we neglect armature resistance $R_a$ (and hence Cu loss), then power developed ($P_d$) by an alternator is equal to the power output ($P_{out}$). Fig. shows the phasor diagram of the salient-pole synchronous generator. The per phase power output of the alternator is

$$P_{out} = P_d = V_t I_a \cos \Phi \ldots \ldots \ldots \ldots (i)$$

Now, \[I_a \cos \Phi = I_q \cos \delta + I_a \sin \delta\]

$$\therefore P_d = V_t [I_q \cos \delta + I_a \sin \delta] \ldots \ldots \ldots \ldots (ii)$$

if $R_a = 0$

then $E_f = V_t \cos \delta + I_a X_d$, \[I_a = \frac{E_f - V_t \cos \delta}{X_d}\]
Putting the value of $I_d$ & $I_q$ in equation (ii),

$$P_d = V_t \left[ \frac{V_t \sin \delta}{X_q} \cos \delta + \frac{E_f - V_t \cos \delta}{X_d} \sin \delta \right]$$

$$P_d = \frac{E_f V_t}{X_d} \sin \delta + V_t^2 \left[ \frac{\sin \delta \cdot \cos \delta}{X_q} - \frac{\sin \delta \cdot \cos \delta}{X_d} \right]$$

$$P_d = \frac{E_f V_t}{X_d} \sin \delta + \frac{V_t^2}{2} \sin 2\delta \left[ \frac{1}{X_q} - \frac{1}{X_d} \right]$$

If saliency is neglected, i.e. for a cylindrical rotor ($X_q = X_d = X_s$),

$$P_d = \frac{E_f V_t}{X_d} \sin \delta$$